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Research Report

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Introduction

The aim of the *Added Value* project is to create a change in teaching mathematics and STEM subjects in European schools using a design thinking approach. The project is run by the *School with Class Foundation* (Poland) together with *NHL Stenden* (Netherlands), *Smilemundo Association* (Spain) and *Universal Learning Systems* (Ireland) and financed by the European Commission from the Erasmus + program. The general objective of the project is to promote innovative mathematical learning strategies and techniques in which teachers become guides, showing their students the real world that can be described and better understood with the use of mathematical tools. This world starts at school, continues at home with its everyday phenomena, and expands to local and global issues that nowadays are within our reach at all times. Mathematics (and associated scientific method) is part of our lives and we in turn are part of the mathematical universe. The project is designed to show the practical side of mathematics to students, to put mathematics in everyday contexts and to make approaches grounded in mathematics (and associated STEM frameworks) accessible and meaningful to students and educators alike. Above all, the project seeks to empower students to believe that they can successfully learn mathematics and to integrate the reasons to do so.

Added Value commenced work in January, 2018. The first phase, Research and Analysis, has been ongoing since then and has focused on two main areas:

- *Secondary research and data gathering*
- *Primary research surveys with teachers and recognized actors in this area*

In addition to providing a research investigation in mathematics teaching and best practice, project partners completed a number of tasks:

- They compiled a profile of the situation of mathematics teaching in their respective countries

- They contributed to gathering a database of international research on this topic
- They provided examples of innovative programmes, particularly in the area of problem-based learning and real-life applications
- They also contributed to developing a database of international experts and expertise in mathematics and STEM approaches and teaching.
- Following this, partners conducted surveys through a common format, and summarised their results for this report.

Background Research Overview

Conducting initial research and assessment of needs among the various actors related to the Added Value project was designed to provide appropriate information for the various research questions required. These questions are critical for the successful development of the project. This process identified efforts active in various settings who are responsible to develop curricula and methods that aim for enhanced teaching and learning in mathematics. A clear expectation was that where there is innovative practice in one or more countries, this may contribute to practice in other partner countries. The shared investigation and learning produces a synergy in which learning can occur across contexts and borders. This underlines identified elements in innovative best practice (which were further expected to provide a basis for the week of learning in the second project meeting in 2018).

The partners, led by ULS, have each conducted secondary/desk research. They have also undertaken primary research in their respective countries. The project has already encompassed a number of important actions intended to:

- Develop research and a research report
- Run a focus study in each partner country
- Interview educational experts and relevant actors
- Run pilot programs with toolkits in schools – at least one in each country.

There were a number of considerations in commencing the research which were discussed and reviewed at the first project meeting. These include:

- Identifying models of best practice in the teaching of mathematical and related subjects (STEM)
- Determining the role of mathematics at a time of change
- Examining teaching methods such as problem-based learning
- Design based learning
- Using mathematics in applied situations.

The need to identify a clear focus among partners on the path to successful outcomes highlights some questions. For example, does the project cover any learning experience that incorporates mathematics? A suggested definition made by the partners is that the *Added Value* project is examining 'Sustainable Mathematical Competence and Numerical Literacy".

Another question is that of clearly identifying the need that prompted both this research and project. Intellectual Output (IO) 2, *Developing the Toolkit*, will require buy-in from the teachers who will be delivering the pilot phase in IO 4 - and ultimately the final Toolkit - and also the additional materials and resources to be developed under IO 3. For both these Outputs to be well received, utilized and deployed in classrooms, it is essential to meet the real needs of those who are expected to deliver mathematical education.

The **Needs Analysis Phase** can take quite a long time. To try to overcome issues associated with activities which are spread over long periods (pressure of other work, etc.) two levels of needs assessment were proposed. The first level built on the areas identified in the project application phase and redefined those using already existing data from the identified contexts. This data, combined with interviews with key personnel, provided the initial overview of key issues and challenges in teaching mathematics, and the possible means to make this more engaging for pupils.

The findings from this first level analysis have provided a focus for the second level analysis. This aims to examine in detail the needs, insights and problems experienced by people within the project partnership, using specific tools such as questionnaires, surveys or group meetings. Other advantages of using two levels of analysis is that after the first level of analysis, it is possible to undertake some reflective actions.

The success of this phase depends on a range of issues. Most important of these is the participation of individuals in the data gathering exercise. To ensure people participate, they need to be well briefed about the project and its activities, why it is being undertaken and how they stand to benefit from it. The exact nature of participation will depend on the setting. Open communication ensured that potential difficulties and conflicts were avoided. Issues such as confidentiality, anonymity and security needed to be addressed with the appropriate stakeholders for several reasons. This guarantees that the information gathered from participants is reliable.

The age cohort of the children being targeted is that of late primary and early secondary, which ranges approximately from 9 -14 years old.

The themes identified thus far by the project group for the toolkit include focusing on problem based learning and real life applications. The extent to which these themes are currently in use can be examined through the research.

Each Partner completed a profile of the situation of mathematics and the curriculum in their own countries. The summary of these can be located under the country below.

Primary Research Overview: Teachers' and Experts' Surveys and Interviews

Following consultation with partners, the format for the surveys was developed. This consisted of number of sections were devised for relevant information and partners conducted the surveys in their own country and also with a number of international experts. The detailed results of the survey from each partner follow. The format for the survey headings was as follows:

Section 1: Country Context

Section 2: Schools

Section 3: Teachers

Section 4: Pupils

Background and Primary Research in Each Country

Mathematics Education in Ireland

Primary Schools

The Primary School Mathematics Curriculum (PSMC) for Junior Infants to Sixth Class was introduced in 1999 (Government of Ireland, 1999). It comprises five strands: Number; Algebra; Shape and Space; Measures; and Data. Spanning the content are skills that pupils should develop, such as implementing, problem solving, communicating, and reasoning. In July 2011, the Department of Education and Skills launched a strategy document, *Literacy and numeracy for learning and life: The national strategy to improve literacy and numeracy among children and young people 2011- 2020* (DES, 2011). In this document, a range of measures designed to raise standards in literacy and numeracy, from early childhood to the end of second-level education, is outlined. These include increased allocation of teaching time to mathematics in schools and greater emphasis on mathematics in the initial education of teachers.

In *Primary school curriculum: Introduction* (Government of Ireland 1999), it is claimed that the curriculum incorporates current educational thinking and the most innovative and effective pedagogical practice. It was heralded as a curriculum that set out clearly not only *what* the child should learn but *how* the child should learn most effectively (*National Council for Curriculum and Assessment* (NCCA) 2000). For instance, the curriculum emphasizes the importance of the children's own experiences as key reference points in learning mathematics. It advocates that children be encouraged to use a range of forms of recording mathematical activity including, as and when appropriate, the traditional written algorithms. The key role of discussion is

emphasized and the necessity for children to work with materials individually and in small groups is outlined.

The areas that are felt to pose the greatest challenge for teachers include:

- engaging children in problem solving and in the general processes of mathematics
- developing the mathematical understanding of children from diverse language backgrounds
- supporting children experiencing difficulty with mathematics
- documenting children's mathematical learning.

There is a great emphasis on children being the instruments of their own learning. The constructivist approach is central to the mathematics program, where children must construct their own internal structures. They are encouraged to develop their own mathematical strategies for solving problems by using their knowledge of one area to explore another and enhance their growth of reasoning. Access to concrete materials is considered necessary for pupils at all class levels from infants through to sixth class. Language also plays an important role. Children need to develop the ability to listen, question and discuss as well as to read and record. Discussion can be in pairs, groups or among the class as a whole. The teacher supplies mathematical language when necessary to enable children to build up an appropriate mathematical vocabulary. Concepts need to be adequately developed orally before children record them in writing, using symbols and mathematical expressions.

From the perspective of developing pedagogy in primary classes in Ireland there appears to be a need to address, with or for teachers, the contextual factors that appear to impinge so much on teachers' efforts to implement appropriate pedagogical strategies. These include factors such as class size and the organization of multi-class teaching where it involves four- and five-year-old children. Teachers in this study clearly indicated that for them, the demands of consecutive or multi-class teaching often necessitate extensive usage of mathematical workbooks. This appears to imply

that in these situations there is consequently less discussion and talk about mathematics, and less active mathematical learning by the children.

It appears that teachers would benefit from in-service work which focuses both on the role of talk and discussion in mathematics pedagogy, but also on the benefits from the perspective of mathematical learning that can arise from discussion. Working from the principal of starting with what teachers indicate support for, then number games and stories seem a good starting point with teachers to help them build interest in children's talk and develop their skills in listening and understanding.

The mathematics component of the Primary School Curriculum is for all children from Junior Infant classes (pre-primary) to Grade 6. The mathematics curriculum aims to help all children to:

- Develop a positive attitude toward mathematics and to appreciate its practical applications in life
- Develop problem solving skills and the ability to use mathematics in everyday life
- Use mathematical language effectively and accurately
- Understand mathematical concepts and processes at a level commensurate to their development and ability
- Become proficient in fundamental mathematical skills and in recalling basic number facts.

In Grade 4, the curriculum is presented in five areas, known as strands:

1. Number
2. Algebra
3. Shape and Space
4. Measures
5. Data.

The strands are interrelated, such that student understanding in one strand is dependent on and supportive of ideas and concepts in other strands. The strands are divided into strand units in which student learning is described using content objectives. Unlike the rest of the Primary School Curriculum—in which subject learning

content is categorized at four levels, each of which consists of a two-year grade band—the content in the Mathematics Curriculum is specified in single year grades.

Table 1 shows the curriculum strands and strand units for Grade 4 and provides some specific examples of the types of skills students are able to develop through their mathematical work. These include:

1. Applying and Problem Solving
2. Understanding and Recalling
3. Communicating and Expressing
4. Integrating and Connecting
5. Reasoning and Implementing.

Table 1: Summary of Mathematics Curriculum for Grade 4, and Sample Skills

Strand	Strand Unit	Mathematical Learning Objectives
Number	Place value	Round whole numbers to nearest 1,000
	Operations	Solve word problems involving adding and subtracting within 9,999
	Fractions	Solve problems involving fractions
	Decimals	Order decimals on the number line
Algebra	Number patterns and sequences	Explore, recognize, and record patterns in number, 0–9,999; describe sequences
	Number sentences	Translate a one-step word problem into a number sentence, and solve
Shape and Space	2-D shapes	Identify, classify, compare, draw, tessellate, and make patterns with 2-D shapes
	3-D shapes	Identify, classify, and construct 3-D shapes Describe relationship of 3-D shapes with constituent 2-D shapes

	Symmetry	Use understanding of line symmetry to complete missing half of a shape, picture or pattern
	Lines and angles	Describe intersecting lines and their angles and classify angles as greater than, less than, or equal to a right angle
Measures	Length	Add, subtract, multiply, and carry out simple division of units of length (m, cm, km)
	Area	Estimate, compare, and measure the area of regular and irregular shapes (cm^2, m^2)
	Weight	Add, subtract, multiply, and carry out simple division of units of weight (kg and g)
	Capacity	Add, subtract, multiply, and carry out simple division of units of capacity (l, ml)
	Time	Work with times and dates; add and subtract hours and minutes
	Money	Add, subtract, multiply, and carry out simple division of money (euro and cent)
	Representing and interpreting data	Use data sets
Data	Chance	Identify and record outcomes of simple random processes

Second Level

There have been significant changes in Irish second level mathematics education in the past decade with the introduction of a new mathematics curriculum entitled *Project Maths*. This major initiative in mathematics education was introduced as an attempt to improve students' real understanding of mathematical content and help them to engage more in problem solving activities. The shift in emphasis to the style of teaching and learning which is advocated in *Project Maths* was partially motivated by the well-documented decline in students' basic mathematics skills on entry to third level education over the past two decades. Current research is

concerned with investigating the effect, if any, this new curriculum is having on third level mathematics education.

A revised syllabus for Junior Certificate Mathematics (lower secondary level) was introduced on a phased basis in September 2010 as part of the *Project Maths* initiative. The syllabus comprises five strands:

- Statistics and Probability
- Geometry and Trigonometry
- Number
- Algebra
- Functions.

In September 2014, all students at the lower secondary level engaged with all five strands of the revised syllabus for the first time. The syllabus is set out in strands to provide continuity with the primary school curriculum. Teachers are encouraged to teach mathematics in contexts that allow learners to see connections within mathematics, between mathematics and other subjects, and between mathematics and its real-life applications. In this way, students can achieve the objectives of lower secondary mathematics and develop proficiency in the following areas of mathematical competence:

- *Conceptual understanding*—Comprehension of mathematical concepts, operations, and relations
- *Procedural fluency*—Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *Strategic competence*—Ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
- *Adaptive reasoning*—Capacity for logical thought, reflection, explanation, justification, and communication
- *Productive disposition*—Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance, and one's own efficacy

Problem solving (i.e. engaging in a task for which the solution is not immediately obvious) is integral to the lower secondary mathematics classroom. The syllabus stipulates that problem solving should not be met in isolation; rather, it should permeate all aspects of the teaching and learning experience. Problems may comprise purely mathematical matters or an applied context. In a mathematics problem solving environment, students are expected to:

- Make sense of the problem
- Make sense of the mathematics they can learn and use when doing the problem
- Arrive at a correct solution to the problem.

In the lower secondary mathematics classroom, teachers focus on helping students to develop mathematical knowledge and skills through the process of solving problems, rather than on helping them to find solutions. They prioritize generating class discussion and facilitating mathematical reasoning as students engage in problem solving. Students learn to analyze problems and break them down into manageable steps, to reflect on their strategies and those of others, and to adjust their own approach where necessary.

Teachers play an important role in helping students develop these kinds of skills. By choosing tasks that present learners with a challenge, they activate learners' mathematical thinking processes, as opposed to imitative thinking processes. By encouraging them to share, explain, and justify their problem-solving strategies, those that work as well as those that do not work, teachers can help learners to develop robust and deep mathematical understanding as well as confidence in their mathematical ability.

International comparative Analysis of Irish Mathematics Curriculum

In 2003, the overall performance of students in Ireland on PISA 2003 mathematics was close to the OECD country average. Students in Ireland ranked 17th of 29 OECD countries, and 20th of 40 participating countries.

This performance may be contrasted with Irish students' performance in PISA 2003 reading literacy and science, for which mean scores were above the corresponding OECD country averages. Several factors may explain the performance of students in Ireland. These include the relative unfamiliarity of students with some of the contexts and processes (competencies) underlying PISA mathematics, and the motivation of students in a low-stakes assessment such as PISA (though there is no evidence that this was a problem for reading literacy or science). It might also be noted that there are substantial aspects of the Junior Certificate syllabus (including Algebra and Geometry) that are assessed only to a small extent or are not assessed at all in PISA mathematics. This implies that students in Ireland did not have an opportunity to display their full range of knowledge in these aspects of mathematics.

In 2010 *Project Maths*, the radically new second-level program of mathematics education, was introduced to all second-level schools in Ireland (full implementation of the project was completed in 2015). According to official sources, the project 'involves empowering students to develop essential problem-solving skills for higher education and the workplace by engaging teenagers with mathematics set in interesting and real-world contexts' (NCCA, 2015). It replaces previous mathematics programs whose foundations lay in the 'New Maths' system introduced in the early 1960s (NCCA, 2002) where the curriculum involved formal 'comparatively abstract and symbolic mathematics' (Oldham, 1991, p. 127) and was 'characterized by emphasis on structure and rigour' (NCCA, 2002, p. 4).

However, *Project Maths* is not a mini-PISA and the two programs diverge in their approach to content and in their commitment to formal, abstract mathematics. In particular, *Project Maths* retains the strand/topic structure in the area of content while the assessment of *Project Maths* reveals the existence of a curriculum focused in part on a traditional approach to abstract, symbolic mathematics, while at the same time prioritizing the interpretation and solving of mathematics problems embedded in realistic contexts. In terms of pedagogy, teaching via problems is advocated by the

DES despite the fact that almost half of the time/marks allocation on the assessment is for traditional abstract mathematics which teachers have long taught by didactic methods rather than active ones.

Ireland has maintained its very high standard in reading and has improved slightly on its performance in mathematics compared with other PISA cycles in the most recent PISA results (2015). In science, Ireland maintained its position of performing above the OECD average, while experiencing a slight score reduction when compared to 2006, the last year when science was examined as the major domain in PISA. However, Ireland's 2015 performance in science marked a more significant drop when compared to the score recorded for 2012.

The key findings of the 2015 report include:

• In reading, Irish students ranked 3rd out of 35 OECD countries, 2nd among EU countries, and 5th out of all countries participating in PISA 2015

• In science, Irish students ranked 13th out of 35 OECD countries, 6th among EU countries and 19th out of all countries participating in PISA 2015

• In mathematics, Irish students ranked 13th of 35 OECD countries, 9th among EU countries and 18th out of all countries participating in PISA 2015.

Science has been a compulsory subject for all primary school pupils since 1999. In the *Primary School Science Curriculum* (PSSC), emphasis is placed on the development of scientific content knowledge (in biology, physics and chemistry) and on the development of scientific skills.

There are four content strands in the PSSC:

1. Living Things
2. Energy and Forces
3. Materials

4. Environmental Awareness and Care.

Although they are included in the PSSC, some elements of Earth Science are included in the Primary School Geography Curriculum (PSGC).

While acknowledging that some excellent science teaching is taking place in primary schools, concerns have been expressed about Primary Science with regard to:

- The amount of time allocated to Science
- The limited Science background of Primary teachers
- The amount of time spent on Science in the initial education (ITE) of Primary teachers
- The lack of appropriate Science-based CPD for Primary teachers.

STEM Curricula in the Irish Education System

Sector	Area	Subject
Primary	Science	Science
	Technology	Although not a curriculum subject per se, use of ICT, as a means of enhancing teaching and learning, is promoted across the primary school curriculum
	Mathematics	Mathematics
Junior Cycle	Science	Science
	Technology	Technology, Material Technology (Wood), Metalwork
	Engineering	Technical Graphics
	Mathematics	Mathematics
Senior Cycle	Science	Biology, Chemistry, Physics, Agricultural Science, Physics & Chemistry
	Technology	Technology, Design and Communication Graphics
	Engineering	Engineering, Construction Studies

Mathematics	Mathematics, Applied Mathematics

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Ireland: Interview Results

Discussions, both formal and informal, were held with a number of maths teachers and related educational experts in this field. The focus was on the later stages of primary school and the early years of secondary, covering the age groups decided by *Added Value* partners.

Section 1: Country Context

1. **What is your opinion of the Maths curriculum in your country?**

The general opinion is that it is vastly improved from systems in place in the past. The introduction of the new primary curriculum in mathematics in 1999 is generally considered a landmark development. The new curriculum and the *Project Maths* structure has vastly improved the teaching, quality of materials and approach in the opinion of all respondents. In 2011, the Department of Education and Science launched a new strategy document (Literacy and Numeracy for Learning and Life). This combines a multidisciplinary approach with a focus on using mathematics n linked ways. It was strongly pointed out, however, that this is not yet fully implemented. The ideas around increased time and resources and advanced training for teachers are strongly welcomed but others feel this is not being sufficiently supported by the required dedicated resources.

2. **What are the positive aspects?**

A new and fresh approach that is flexible and creative. Mathematics is seen as part of a general thrust towards numerical competency and familiarity. Understanding, problem-solving strategies and methods of learning maths are all felt to be significantly improved. Many smaller initiatives have developed to embed maths and to develop a culture of shared mathematical ability. The primary curriculum is also linked to other subjects in often creative ways. In smaller schools this has a particularly

positive learning benefit in terms of linking topics and areas of knowledge. The general philosophy and strategy are strongly endorsed by all respondents.

3. **What are the negative ones?**

Performance levels are still inadequate. Maths is perceived as a difficult and inaccessible subject. Despite many efforts at reform and improvement, basic mathematics skills on entry to third level education over the past twenty years have stagnated and even deteriorated. The primary curriculum is acknowledged as creative and exciting, however class sizes at primary level are large in many schools, which mitigates against effective implementation of the curriculum. In some schools this is addressed by having Resource Teachers for Maths.

The senior secondary is still dominated by an examination system that relies on memory and repetition rather than initiative in solving problems. Skills are based on examination preparations and not independent competence. A critical issue for respondents therefore is assessment and the various methods that would support learning. Assessment is deemed as far too rigid at senior cycle. There is consistent concern that verbal commitment to innovative learning and change in mathematics needs to be matched by practical alterations in assessment methods, teacher training resources, teaching materials, new pedagogies and vastly enhanced in-service training initiatives.

4. **Do you have the option to deliver the curriculum in your own way, for example are there proscribed class times, restrictions due to state examinations?**

The primary curriculum is relatively flexible. Broad learning targets and specified learning outcomes are indicated but flexibility is permitted up to the limits specified by the NCCA and the Department. A comprehensive system of school inspection is designed to ensure standards and teaching and content quality. This is generally valued. However, the continued reliance of terminal examination systems at the senior secondary cycle is universally accepted as radically different in both approach and effect The Junior Certificate examination has been finally modified but not

without massive opposition and resistance. The Department's preference to move towards a system of continuous assessment has been met with ferocious opposition by many teachers, administrators and teaching unions. The rigid examination system is said to favor neutrality and objectivity and avoid bias in assessment. Most accept, however, that the system is exhausting, counterproductive, reliant on rote memorization and inimical to skill transferability and independent critical thinking. The system of the Leaving Certificate examination has not altered since 1922. Proscribed class times are laid down for all schools in the Irish system.

Section 2: Schools

1. **How does your school support mathematics and related subjects?**

Mathematics is a priority subject in all schools. The curriculum regards maths as an essential element in the core subjects in all schools. Mathematics has been prioritized in the new curriculum and the most innovative development has been the creation of Project Maths. Schools try to prioritize creative ways to improve the perception of maths. This varies in primary schools, depending on school size and location. In the senior cycle, the subject is shaped by the examination system. Linkage to other related subjects (sciences and technology) has been generally welcomed.

2. **Do school timetables provide sufficient time for mathematics?**

Generally, yes. Times are allocated for all schools. Class sizes in primary school affect the time available. The system in Ireland is for a single teacher per year in primary school, while secondary schools have a teacher per subject, with proscribed times allocated to each subject. Class sizes are usually smaller in secondary school.

Do schools provide adequate facilities and resources for the teaching of maths and sciences? For example, is it equipped in science labs? If so, how are they organized? What kind of access students have (only during the classes, afterschool etcetera?). Which tools can be find in the labs?

This varies significantly depending on the school size, type, location and tradition. Ireland has a divided system of schooling, which becomes acute at senior secondary level. Public schools exist together with private schools and a newer system of ETB schools (Education and Training Boards) which emerge from the older vocational (technical) education systems. These various strands had significant discrepancies in terms of resources. This legacy persists. Some of the larger and well-endowed private schools have excellent facilities and ample resources. One such school in this survey is able to give each pupil an iPad and has fully equipped science labs and facilities, with strong supports for mathematics. Other schools had minimal resources. All have prescribed textbooks supporting the national curriculum. Some schools have seen afterschool programs and voluntary clubs such as *CoderDojo* across the country. There is significant variation in resources and tools available in science labs.

Facilities and resources in primary schools can vary from area to area. Extra supports are given to schools in designated areas of disadvantage, known as DEIS schools. Schoolbooks and workbooks are a cost on parents [although supported in DEIS schools]. In recent years, there have been moves towards addressing the issue of costly books, through school libraries, and also through using more reusable workbooks and materials, but it is still an annual contentious topic in Irish education.

3. **What is the level of autonomy afforded to teachers to deliver mathematics programs?**

At primary level, there is more autonomy. By senior secondary cycle this is severely restricted because of the demands of the State examination system. In general, the Irish education system is quite centralized and the department exerts a significant influence on standardization to maintain consistency and quality. Various schemes exist to promote experimentation and several pilot programs are used to facilitate creative and fresh approaches. Individual autonomy is not strongly developed however.

Section 3: Teachers

1. **What kind learning of materials are used most often in classroom (books, websites, mobile apps)?**

Primarily prescribed textbooks. Many schools now have good technological resources and most are connected with interactive whiteboards for example. There is significant variation in some areas but generally the primary system has developed a deep and important level of access to digital resources and online materials. The secondary system lags behind. In the senior cycle, if it is not connected to the specified state examination, and the associated prescribed textbooks, it is not used. The textbook still rules. There is a lack of awareness and familiarity with advanced technologies and how they can be used in teaching and learning. Most schools ban mobile phones or tablets for students. In recent years, Irish teachers developed a website to support the curriculum called Mathletes.ie, linked to the Khan Academy model http://mathleteschallenge.khanireland.org/. This was piloted and became so successful that Ireland is now a co-founder with the Khan Academy of Learnstorm, a global maths challenge. Other examples include: IXL Maths Practice; Symmetry School: Learning Geometry; MathWorkout. Teachers can find support and resources through www.scoilnet.ie.

2. **Do you have any preferences? Please name one favorite tool that you use or know is used.**

Responses varied considerably. In our response group, the strongest preferences were for web-based courses, materials and exercises. These were valued highly.

3. **What styles of teaching are offered? Describe the interactions in a typical class.**

Direct instruction is the normal method. Reliance on the prescribed textbook is supplemented by teaching methods which promote group work, problem solving and interaction, primarily at the primary or junior levels. At later stages, traditional didactic approaches are the norm.

4. **What are the typical teaching methodologies currently employed by you?**

Direct instruction, Group work. Interactive sessions. Problem solving in teams.

5. **Which of these motivates pupils the most? Please give concrete examples: games, experiments, artistic performances, platforms, books etcetera. If there are any particularly useful links/site/resources please provide.**

Pupils respond well to games. But these are also seen as 'once off' and the sustainability of learning outcomes from the method is questioned. On the other hand, more and more students are technologically proficient and may employ mathematics in deep and positive ways, often without even knowing or acknowledging that they are so doing. Games also have risks, especially for boys who have big experiences of computer based gaming from home. Connecting games to learning is not always easy, but certainly possible. Challenges and contests are usually good. One teacher mentioned the game 'Zoombinis' which she had found very useful. Another activity that has proved successful in some primary schools has been *Maths Stations*, where a particular aspect of maths is highlighted at a station, with a champion to explain it; students move from station to station depending on their gaps in knowledge. The availability of time to engage in games and exercises is often an issue. The Irish curriculum is quite specific about time allocation and variations from agreed schedules can be problematic. This becomes acutely pressing at secondary level as the impact of and preparation for State examinations becomes more intense. An emerging interest is robotics and coding. Many times, this may be done after school or in extra time.

6. **Do teachers use problem-based learning?**

Some do, especially in primary and junior schools. There is awareness of STEM approaches and an openness to connect the thinking and methods. Again, this is tied to available resources and recognition of teachers' initiatives in taking leadership.

7. **Do you have the opportunity to develop lessons based on real-life applications?**

Occasionally. Not consistently. In an uneven manner. Constant reference to available resources, support and time.

8. **What examples are there of real life applications for mathematics teaching?**

Anything to do with money – budgeting, selling, buying, accounting. The use of commerce such as in shopping or bargaining (e.g. percentages). Measuring and construction. Cooking, recipes, baking. Sports and games – recording statistics and measuring outcomes in terms of performance.

9. **What links do you as mathematics teacher perceive with teachers of other subjects? Which subjects are these?**

Strong links described with science, especially physics and/or engineering. Other links with social sciences, geography, environmental studies and cookery. Strong links to construction in general and architecture in particular. In junior cycles and in smaller schools one teacher may teach several subjects or, in fact, all subjects. Interaction and interplay between subjects may therefore be more common.

10. **What is the state of teachers' relationships with their pupils?**

In Ireland, this is generally good and positive.

11. **What Domains of mathematics and science & technology are you as a teacher particularly interested in?**

Various opinions expressed. Centered on technology, computers, geometry and environmental sciences. As an agricultural country, students and school communities have a strong interest in meteorology, earth sciences, ecology, and farming. Pervasive interest emerging in artificial intelligence issues. Several mentioned astronomy and cosmology. Some mention made of marine biology and oceanography.

12. What would be the main challenges in these domains/subjects?

Resources, materials, up to date materials, accessing the vast amount of web-based materials. Resources and facilities to undertake experiments. For certain subjects like astronomy (which is not formally taught in Ireland) access to telescopes. A noted area of frustration is lack of good research materials to explore options.

13. What gaps would you perceive in capacity and strengths among students?

This was left largely unanswered apart from a general concern that many do not have independent learning skills. The negative impact of brutal terminal examination systems and resultant expectations on memorization were universally mentioned.

14. What and where could extra support be provided?

Depends on student needs. In recent years, there is a strong national emphasis on inclusive education and access. This has posed many challenges in terms of universal design for learning on the one hand and the accommodation of diversity on the other. This is a major national imperative in Ireland with a significant increase in diversity at all levels.

15. What kind of extra challenges might be included to engage and stretch students?

Creative problem solving and challenges that engage the interest and imagination of students. Any other creative or innovative games. Things based on the local community or identified characteristics (measuring, observing, recording, tracking, etc.). Social sciences and facts mentioned. Comparative analysis of common elements. Anything that promotes critical or reflective thinking and practice.

16. **This project plans to develop a toolkit for teachers, what would be useful to you in such a toolkit?**

Resources, pointers to other resources and relevant creative materials, problem-solving kits, challenges and quizzes, games, web-based resources, apps. Guidance on better practice. Supplementary exercises or quizzes or innovative concepts and practices. Templates for learning exercises.

Section 4: Pupils

1. **What is the state of teachers' relationships with their pupils?**

In Ireland, this is generally positive. The existence of strong community bonds and links in a relatively small country produces many benefits. Communication is intense and fluid. At the primary level this is strongly present, especially with smaller schools in rural areas. A general perception is that mutual respect, shared mission and commitment are key characteristics of student/teacher relationships in Ireland.

2. **What Domains of mathematics and science & technology is particularly interesting for your students?**

Games, puzzles, mysteries in science to be solved, common questions about how things work, especially in science and technology. There is not deep interest in computer programming per se, but huge interest in how ICT and digital resources can shape things and demonstrate functionality. Once interest is generated, students will passionately follow mathematical lines of enquiry – whether in life sciences or operations of machines. A common interest in astronomy, space and other issues connected with space exploration or exploration of the universe. Others are deeply interested in ecological sciences and sustainable energies. A noted element for many was the impact of technological approaches to social issues and questions such as demographics, poverty, hunger and/or migration. Another ongoing area of technological interest is anything to do with communications.

3. **Which of the learning methodologies motivates pupils the most? Please give concrete examples: games, experiments, artistic performances, platforms, books etcetera. If there are any particularly useful links/site/resources please provide.**

Games and puzzles are always useful. Many resources are available on the website of the Department of Education and Science (*ProjectMaths* curriculum website) e.g. http://www.projectmaths.ie/for-teachers/professional-development/learn-to-use-geogebra/

Other cited resources in Ireland include:

a) Regional Centre for Excellence in Mathematics Teaching and Learning: University of Limerick
b) Department of STEM Science, Technology, Education and Mathematics, Mary Immaculate College
c) Maths exercises aligned to Irish curriculum – www.ixl.com
d) Quality maths resources – www.classmaths.com

4. **Do you think that your pupils see STEAM subjects as useful in everyday life?**

If this is explained and discussed, yes.

5. **What feedback on STEAM do you receive from your pupils?**

This is extremely difficult to answer. STEAM or STEM is not taught specifically in Ireland. Informally, STEM approaches are used. But in the absence of a formal or standardized methodology or curriculum stream, it is impossible to isolate STEM specific responses. In initiatives monitored by the STEM experts in Stranmillis College and St Mary's in Belfast, responses are highly positive.

INTERVIEWS

1. Elizabeth Oldham (Alexandra College; Trinity College Dublin)
2. Trudie Abrahamse (Alexandra Junior School)
3. Erol Mustafov (Alexandra College)
4. Paul Flynn (CBS Westmoreland Street)
5. Ciaran Lynch (Thurles LIT)
6. Aisling Bruce (DALC Dublin)
7. Geraldine O'Connor (CICE, DCU)
8. Philomena Cherry (Rosmini College)

Existing Research findings on Maths Teaching: Ireland

Ireland has been deeply involved in innovative mathematics teaching in recent years. This is greatly aided by the work of the *Department of Education and Science*, the new curriculum promoted by NCCA, various EU funded initiatives and innovative projects and grassroots movements among teachers and parents. The huge emphasis on STEM approaches relates to the preponderance of US multinationals active in ICT sector and pharmaceuticals. The following indigenous resources have been identified:

1. http://www.classmaths.com/ Classmaths is a Galway based, Irish website dedicated to providing top quality mathematics resources to primary school aged children. The website works by linking kids to fun games based on their chosen aspect of their maths curriculum. Games cover basic addition, subtraction, multiplication and division as well as algebra, shapes and code.
2. http://www.ixl.com/ IXL focuses on encouraging children to have fun while learning mathematics. The website is directly aligned to the Irish primary, Junior Cert and Leaving Cert curriculums, offering endless practice problems and exercises. The site caters for students at all levels, ranging from junior infants up to fifth year. IXL offers kids a chance to learn basic and more complicated maths skills in a fun and exciting way. Site membership fees apply (€59.00 annually).
3. http://www.projectmaths.ie/for-teachers/professional-development/learn-to-use-geogebra/ Projectmaths is a resource for teachers of mathematics. It provides workshops for newly qualified mathematics teachers. It also provides teacher supports, professional development, access to Geogebra and MATA, maths and STEM resources in the Irish language.
4. The *Regional Centre for Excellence in Mathematics Teaching and Learning* is based in the University of Limerick and facilitates in-service training as well as best practice development.
5. *Mary Immaculate Teacher Training College* officially launched a new department entitled the Department of STEM (Science, Technology, Education and Mathematics) Education. This new departure comes at an exciting time in the Irish educational landscape and responds to the *National Strategy for*

Education (2011) which challenges third level providers to set out the distinctiveness of their respective institutions. The STEM Education Department will contribute towards producing STEM-literate citizens – a goal that is driven not only by economic imperatives but also from a social justice perspective. According to Dr. Aisling Leavy, Head of STEM Education Department, this department will bring together a wealth of expertise and innovative practice from Science Education, Mathematics Education and ICT/Digital learning. To that end over 5000 primary school children with their teachers and families attended free STEM-related activities, organized by the Department of STEM Education, and hosted seminars in both MIC's Limerick and Thurles campuses. According to Professor Eugene Wall, President (Acting) of MIC, Mary Immaculate College has an established track record in encouraging children's long-term interest, understanding, knowledge and skills in STEM subjects – and developing primary school teachers' confidence, knowledge and skills in the teaching of STEM. "Science education is crucial in fostering wonderment in children and in helping them to develop scientific reasoning. To that end, MIC's STEM Education team maintains a very strong up-to-date knowledge of contemporary developments in their fields by coordinating, managing and conducting evidence-based research – and in developing new innovative, effective teaching and learning methodologies".

Mathematics Education in Poland

Another Reform

Last year the education system in Poland faced significant changes. This was another reform of the system introduced without proper preparation. The reasons for changes were political. Teachers do not identify with the reform and openly criticise it. Previously, pupils attended 6-year primary schools and then continued education in 3-year lower secondary school, followed by 3-year upper secondary school. The new legislation returned to 8-year primary school and 4-year secondary school. This entails changes in the curriculum, including maths.

Tyranny of Tests

Each education level ends with exit exams, e.g. in mathematics. This pressure on test solving skills makes teachers focus on teaching not reasoning and deducing, but skillful test solving.

Purpose of Mathematics Teaching

Polish maths teachers cannot see the big picture: they do not consider what the purpose of teaching mathematics is and what use their students are going to make of it. They perceive goals from an operational point of view. Even the most aware teachers, who believe that mathematics is an inseparable part of the world we live in, are not necessarily able to convey this knowledge to their students. They are fascinated with mathematics and sometimes it is difficult for them to understand that this is not so obvious for everyone else.

‘Mathematics is beautiful. It is beauty in its purest form.’

‘Everything is mathematics. How can anyone not see this?’

Negative Selection for the Profession of Teacher

All surveyed teachers and experts admit that a major concern of the Polish education is negative selection for the profession of teacher. While before 1989 the profession of teacher was prestigious, currently it is not much respected. This is also linked to low salaries. Obviously, there is a lot of enthusiasts in the Polish schools, but they often come across a lot of obstacles, also among their more passive colleagues who would rather preserve the *status quo*.

Cooperation of Teachers

Polish teachers do not have the practice of cooperating with one another. Classes are conducted within the framework of specific subjects. Mathematics, science, chemistry or biology are taught by different teachers. Each of them follows their own curriculum. Sometimes it happens that teachers, on their own initiative, agree upon the order of the topics covered.

Such cooperation happens more frequently among teachers of the same subject. As a result, maths teachers from the same school develop concepts together and sometimes organize a school project like the Pi Day or the Mathematics Day. It happens more and more often, even despite obstacles of a structural nature (the school and curriculum structure itself does not support such cooperation).

Lab Equipment and Tools

Traditionally, maths is taught without any tools. Teachers use whiteboards and markers, sometimes on-line tools (e.g. Geogebra). Some of them encourage students to use their smartphones, e.g. a calculator. However, mobile phones are prohibited in some schools.

Poland: Interview Results

Section 1: Country Context

1. **What is your opinion of the maths curriculum in your country?**

The maths curriculum includes a broad range of topics. According to the curriculum, students should not only master basic skills (e.g. counting), but also acquire logical thinking and problem solving skills. In practice, teachers seem to overlook this requirement set out in the curriculum. Even the most active ones (among those participating in the survey) point out that the school is, in fact, aimed at preparing students to pass tests.

2. **What are the positive aspects?**

Asked about the positive aspects of the curriculum, teachers were unable to name any.

3. **What are the negative ones?**

All teachers and experts share the view that the maths curriculum is overloaded. It includes too many topics. According to half of those surveyed, some topics are introduced too soon. Too much content in the curriculum makes teaching maths a race against time. The system of 45-minute lessons, which are rarely blocked, does not facilitate effective teaching either.

4. **Do you have the option to deliver the curriculum in your own way, for example are there proscribed class times, restrictions due to state examinations?**

Typical teachers are of the opinion that the curriculum imposed on them does not enable free arrangement of content and problem-based teaching, because there is not enough time. Some teachers (half of the surveyed) state, however, that for many of their colleagues this is only an excuse.

'It is easier to conduct a class the same way for years and not come up with anything new.'

On the other hand, a teacher's work is only verified by test results of his or her students. In Polish state schools, teachers enjoy a wide margin of freedom. What they do behind the closed door of a classroom is not controlled by anyone. The most active teachers exercise this freedom. They feel, however, that they 'get around the system'.

Section 2: Schools

1. **How does your school support mathematics and related subjects?**

n/a

2. **Do school timetables provide sufficient time for mathematics?**

45-minute maths classes are conducted usually 4 times per week. Sometimes the headmaster adds one hour to the timetable.

3. **Do schools provide adequate facilities and resources for the teaching of maths and sciences? For example, is it equipped in science labs? If so, how are they organized? What kind of access students have (only during the classes, afterschool etcetera?). Which tools can be find in the labs?**

Classes are conducted in classrooms equipped with whiteboards, sometimes computers and interactive boards. Some classes are conducted in labs dedicated to the specific subject, but it is not a rule. Typically, students only have access to the lab during the class, under the teacher's supervision. A vast majority of teachers conduct classes by following the handbook, which is not interesting for the students.

4. **What is the level of autonomy afforded to teachers to deliver mathematics programs?**

Teachers claim they are under tremendous pressure. The main expectation of the parents is that their children pass school exit exams successfully, and not that they master logical thinking or problem solving. In fact, in state schools no one verifies the way maths classes are conducted. The ultimate and only recognized test are school exit exams. Rarely do teachers exercise the autonomy afforded to them. Afraid of objections from parents or school management, they choose to follow the well-trodden path. Hence, most teachers choose a specific handbook and delivers the curriculum by following it step by step.

Section 3: Teachers

1. **What kind of learning materials are used most often in classroom (books, websites, mobile apps)?**

Handbooks are used more often. Some teachers use on-line tools.

https://www.geogebra.org

http://www.wolframalpha.com

https://photomath.net

The obstacle here, however, is the lack of language skills. Most teachers do not know this language. A vast majority of students owns smartphones, but their use in a maths class is not obvious. YouTube channels are popular among students. Teachers rarely use on-line courses. The most active teachers prepare teaching tools on their own.

2. **Do you have any preferences? Please name one favorite tool that you use or know is used.**

The most valuable tools are physical objects. Teachers often prepare these on their own, but it is time-consuming. Hence, for geometry or even fractions classes, teachers prepare or obtain teaching tools which are physical objects (solids, chocolate divided into pieces).

3. **What styles of teaching is offered? Describe the interactions in a typical class.**

In most cases teachers work using traditional methods. They explain problem at the whiteboard, ask students in front of the whole class, assign individual work to students. The most active teachers sometimes organize team work, although 45-minute lessons are limiting. Teacher-oriented teaching. Student's own motivation is rarely used. Modern methods of education (Montessori, Steiner, democratic schools) are gaining popularity, but they are hard to find in state schools.

4. **What are the typical teaching methodologies currently employed by you?**

Direct instructions mixed with individual work. Less frequently team work.

5. **Which of these motivates pupils the most? Please give concrete examples: games, experiments, artistic performances, platforms, books etcetera. If there are any particularly useful links/site/resources please provide.**

All teachers admit that the major challenge is awakening the students' interest. Thus, the problem presented in an unusual way, elements of a plot line or riddle are an interesting idea. Games, competitions. One of the teachers organizes TED talks on mathematics. Gamification or escape rooms are also come across.

6. **Do teachers use problem-based learning?**

The most active teachers seek to use it. The majority, however, does not teach this way, as it is more time-consuming. There are more and more attempts to apply problem-based learning at schools, organizing such projects, as the Pi Day or the Mathematics Day. Some of these are interdisciplinary projects.

7. **Do you have the opportunity to develop lessons based on real-life applications?**

Teachers have the opportunity to develop lessons based on real-life applications. However, they do not always seize this opportunity, arguing they have little time. Some handbooks facilitate such an approach. The tasks refer to real life.

8. **What examples are there of real life applications for mathematics teaching?**

Planning a renovation of the school canteen, planning the costs of a school trip, calculating the promotional price for tablets (desired by students) in an electronic store, calculating the loan taken out in a bank along with interest.

9. **What links do you as mathematics teacher perceive with teachers of other subjects? Which subjects are these?**

Maths teachers usually work alone to deliver the curriculum. Sometimes, if they have good personal relationships with other teachers, they work together with science

and IT teachers. This cooperation consists mainly in agreeing on the order in which certain topics will be covered, but from time to time teachers also organize joint projects. Sometimes mathematicians work with PE teachers, but everything depends on personal relationships between them.

10. What is the state of teachers' relationships with their pupils?

Almost everything depends on the teacher's attitude. If the teacher is capable and friendly to students, he or she can develop almost partner relationships with them. Traditionally, a teacher in the Polish school had almost official authority, but this model ceased to work over ten years ago. A teacher either earns students' favour, or has to face their defiance.

11. What Domains of mathematics and science & technology are you as a teacher particularly interested in?

Anything that has real-life applications and can awaken the students' interest. It is important that it prepares the students to complex problem solving in practice.

12. What would be the main challenges in these domains/subjects?

Students perceive mathematics as a difficult and boring subject. Teaching this subject with unsuitable methods discourage students and makes it difficult to reawaken their interest and enthusiasm later. Also, mathematics appears to them as detached from reality.

13. What gaps would you perceive in capacity and strengths among students?

Students tackle calculations and schematic solutions fairly well. However, they lack problem solving skills; they are unable to connect the dots and make use of what they learnt from different subjects.

14. What and where could extra support be provided?

There isn`t any extra support.

15. What kind of extra challenges might be included to engage and stretch students?

Above all, we should look for examples which have something in common with the students' everyday life. It does not involve paying taxes or taking out a mortgage. Engrossing forms such as gamification, escape rooms, mystery and competition increase motivation.

16. **This project plans to develop a toolkit for teachers, what would be useful to you in such a toolkit?**

Physical objects which can be used in practice. For the entire class. The tool should be related to everyday life and touch upon different issues included in the curriculum. A scheme or framework so that you can see the larger picture.

It would be best if the tool was linked to different areas of life and divided into categories: world, country, community, home, school. It should be user-friendly and the use should not be time-consuming. It should be visible in the classroom, e.g. on displayed on the wall, and prepared in Polish. It should be possible to use it during a 45-minute lesson.

INTERVIEWS

8 IDI with teachers (on-line); 30-60 minutes

1 IDI with maths educator – expert (on-line); 40 minutes

1 FGI with 3 experts from FSZK; 90 minutes

Existing Research findings on Maths Teaching: Poland

1. There is a social acceptance In Poland of not learning math since it is perceived as being too difficult.
2. Early education teachers lack substantial mathematical knowledge. Teachers who are insecure in the field of topics discussed with pupils limit themselves to only conveying generic schemata.

3. Math tutors teaching in further levels of education possess solid substantial preparation. They often lack, however, practical didactical skills: communicating with pupils, organising work in small groups, creating positively motivating environment and situations. Teachers are aware of the fact that the core curriculum stresses the importance of independent thinking, arguing and creating strategy for problem solving. This is not reflected in practice.

4. An obstacle in mathematical education seems to be, among others, focusing on basic skills rather than on capability of independent thinking or solving non-standard problems. Overly focusing on basic tool skills (on two primary stages of education) kills creativity in pupils.

5. One of the existing teaching methods in use is that a teacher or pupil solves the task on the blackboard and the rest of the pupils observe the process.

6. Communication problems: rather than attempting at understanding of the path that the pupil is following when solving a problem, control of formal correct version of reaching a solution dominates. As a result, pupils stop asking questions and solve mathematic problems less creatively.

7. The inability of organising work in groups. Teachers do not understand this method and how group dynamics might be used. Their pupils often work in groups – on declaration level.

8. Inefficiency of trainings. Trainings finished (e.g. on teacher-pupil communication, team work etc.) do not have effect on teachers' skills, that might be used in practice.

9. Seasoned methods: “IBE mathematical bubbles” - a project for teachers that contains elements of training and coaching, lasting 3-4 years. It delivers support within didactics and psychology to a group of teachers. An important condition is support from school's director and local government.

Mathematics Education in Spain

Educational context in Spain

The regulated teachings offered by the Spanish educational system are:

- **Early Childhood Education** (0-6 years, not obligatory, free from 3 - 6 years)
- **Primary Education** (6-12 years, obligatory and free)
- **Compulsory Secondary Education** (12-16 years, obligatory and free)
- **Bachelor's Degree and Vocational Training** (+16 years, not obligatory)
- **University Education** (+18 years)

About financing, in Spain there are three types of educational centers: the **public** ones (financed by the State and the Autonomous Communities), the **private** ones (by private funds) and the "**concerted"** ones (private centers but subsidized by public funds).

Spanish Primary Mathematics Curriculum

This is organized in **six courses** that are usually followed between six and twelve years. It is taught by teachers with a general education, with competence in all the subjects of this level.

Main information of the Official State Gazette (BOE) nº 52, year 2014 (page 38):

https://www.boe.es/boe/dias/2014/03/01/pdfs/BOE-A-2014-2222.pdf

The subject of mathematics in Spanish primary education is obligatory and is taught in all the years of primary education.

Below are the main key ideas (extracted from the BOE of 2014):

- Mathematics allows us to know and structure reality, analyze it, assess it and finally to make decisions.
- They are a set of knowledge and ways of acting that involve not only using quantities and geometric shapes, but also asking questions, obtaining models and identifying relationships and structures.

- They incorporate deduction, precision, rigor, estimation, approximation and probability.
- There is a double function of learning mathematics:
  a) Useful in other areas (in everyday life, in the workplace...)
  b) Contributes to enhance cognitive abilities of children

Contents

The contents have been organized in five large blocks:

- Block 1. Processes, methods and attitudes in mathematics
- Block 2. Numbers
- Block 3. Measurements
- Block 4. Geometry
- Block 5. Statistics and probability

Methodology

The 2014 Core Mathematics Education Curriculum emphasizes the importance of experiential learning*: "Work in this area in Primary Education will be based on experience; the contents of learning start from the close and should be addressed in contexts of identification and problem solving ".* (Official State Gazette (BOE) nº 52, year 2014).

Likewise, it points in the direction of combining with other subjects, largely through the development of a problem-based learning methodology: *"Problem solving processes constitute one of the main axes of the activity mathematics and should be the main source and support of learning throughout the stage, since they constitute the cornerstone of mathematics education. In the resolution of a problem, many of the basic capacities are required and used: reading, reflecting, planning the resolution process, establishing strategies and procedures and reviewing them, modifying the plan if necessary, checking the solution if it has been found, until the results have been found and communicated,"* (Official State Gazette (BOE) nº 52, year 2014).

Comparison with other countries

The Trends in International Mathematics and Science Study (**TIMSS)** is produced every four years, scoring 49 countries. The highest scoring country in 2015 was Singapore with 618 points, while Chile scored the lowest with 478 points. The Spanish average score in the **TIMSS** was 505 points in mathematics, which is 23 points more than in 2011. This score placed Spain above the average grade for nine and ten-year-old students from the countries analyzed. The result has improved, but still remains below the OECD average of 525 points and the EU average of 519 points.

https://www.mecd.gob.es/inee/dam/jcr:1ce5e042-4ee4-4d8f-8d0b-605586dc0159/educainee50provokk.pdf

Spain: Interview Results

Our focus group consisted of nine education professionals in STEAM, three men and six women. They have all worked or currently work as mathematics teachers in primary (from 6 to 12 years old). Our goal was to create a diverse focus group; therefore, we have four teachers working in Barcelona, two teachers working in Chile using Montessori Pedagogy, a teacher who works from the Waldorf pedagogical approach, another teacher using a pedagogical technique in STEAM and finally an expert who promotes mathematics in Barcelona.

Here are the members of our focus group:

PROFESSIONAL	VOCATION	LOCATION
Bernat Ancochea	Mathematics teacher in primary	Barcelona
Joan Bassas	Mathematics and Science teacher in primary Former School Principal	Barcelona
Laia Ermi	Mathematics teacher in primary	Barcelona
Teresa Montenegro	Mathematics teacher in primary	Barcelona

Karla Pacheco	Mathematics teacher in primary (Montessori pedagogy)	Chile
Marion Acuña	Mathematics teacher in primary (Montessori pedagogy)	Chile
Violeta Moreno	Mathematics teacher in primary (Waldorf pedagogy)	Barcelona / France
Ana Albalat	Civil Servant in STEAM in Generalitat de Catalunya Former mathematics teacher in primary	Barcelona
Claudi Alsina	Mathematics teacher in primary Author of over 60 books on the promotion of mathematics International lecturer	Barcelona

Section 1: Country Context

1. **What is your opinion of the Mathematics curriculum in your country?**

The more experienced teachers in Spain have found a clear consensus that the STEAM curriculum has dramatically improved in the last 20 to 30 years. We can observe a progression from the older curriculum and focus solely on content today, more based on skills and interconnected with other areas. Moreover, the three teachers working for more alternative pedagogies to the official (Waldorf and Montessori) stand out from the official curriculum content in excess and a clear lack of connection with other disciplines.

2. **What are the positive aspects?**

Only in cases of Catalan curriculum (enacted in 2015 and is different than Spanish national curriculum) we have found positive aspects: it is centered on skills rather than content, connected with other areas, focused on a more qualitative evaluation and is focused more on the process of each student.

3. **What are the negative aspects?**

100% of the respondents agree that there is an excess in the content, a lack of connection between their units and that there is not enough importance given to skills

versus content. On the other hand, the teachers that use more alternative approaches complain that it is very intellectualized and there is very little emotional and physical curriculum.

4. **Do you have the option to deliver the curriculum in your own way? For example, are there prescribed class times or restrictions due to state examinations?**

At this point we see a large difference depending on whether they are public or private schools. Respondents working in public schools agree that there is an excess of content and are forced to follow the curriculum precisely, barely having time to stop and discuss concepts in a deeper manner. In comparison, teachers in private schools have greater flexibility to adapt the curriculum. However, the official curriculum and the Catalan decree (119/2015) for primary education leaves great flexibility to the teachers.

Section 2: Schools

1. **How does your school support mathematics and related subjects?**

There are plenty of free courses for STEAM teachers (Generalitat offer) mainly on ICT. Many public schools have science rooms or labs. It should be highlighted that in Waldorf and Montessori schools, teachers receive profound training in STEAM, including a large number of tangible tools for mathematics (especially in the Montessori approach). Most of the other more traditional schools mainly use textbooks as the main source of support for mathematics teaching.

2. **Do school timetables provide sufficient time for mathematics?**

Generally, no, except for those working for projects-based learning, where each student is free to choose the time they want to spend on each subject. Also, we want to emphasize the approach of Waldorf education about leaving breaks between learning lessons in order to later recover the information more organically.

3. **Do schools provide adequate facilities and resources for the teaching of mathematics and sciences? For example, are the schools equipped with science labs? If so, how are they organized and what tools can be found in the labs? What kind of access do students have, such as only during class, after school, etc.?**

At this point we see a variety of resources. In general, many schools have laboratories and microscopes, but only some have 3-D printers. We also want to emphasize the projects that interconnect various disciplines, which often have more resources and personnel than the rest.

4. **What is the level of autonomy afforded to teachers to deliver mathematics programs?**

100% of respondents agreed that they have great autonomy in terms of how they are able to approach the pedagogy of their classes. For example, we found a public school in Barcelona (Patronat Domènech) working entirely without using any textbook in their classes.

Section 3: Teachers

1. **What kind learning of materials are used most often in classroom (books, websites, mobile apps)?**

100% of respondents agree they mostly use manipulative material. Secondly, we have textbooks, and finally, digital materials, which are underused (just 3 of the 9 respondents use digital media).

2. **Do you have any preferences? Please name one favorite tool that you use or know is used.**

Highlights the hopscotch game to learn how to add and subtract, mandalas wool and using various dramaturgies/stories to create a context that makes sense to solve certain mathematical problems. Four of the nine respondents use recycled materials and build their own materials.

3. **What styles of teaching are offered? Describe the interactions in a typical class.**

Six of the nine respondents prefer to focus their classes more individually or in small groups (ideally two to three students maximum), compared to the other three respondents who make their classes larger group and with a more direct style.

4. **What are the typical teaching methodologies currently employed by you?**

100% of respondents agree that it is important to use manipulative methods and to first allow a visual and tangible approach. For example, the center "Gamar" of the renowned pedagogue Maria Antonia Canals, has a wide range of manipulative methods of all types. Along the same lines, it also emphasizes the visual and artistic methodologies Montessori and Waldorf approaches. Also, GeoGebra is a tool that is currently booming and is a complement to traditional manipulative methods.

5. **Which of these motivates pupils the most? Please give concrete examples: games, experiments, artistic performances, platforms, books etc. If there are any particularly useful links/sites/resources please provide.**

Seven of nine respondents agree that manipulative games and table games are highly appreciated by the students. Also, the stories and the use of GeoGebra. Some links of special interest:

- Bank of principal resources of public education in Catalonia:

  http://xtec.gencat.cat/ca/recursos/matematiques/

- Anton Aubanell's resources on Mathematics:

  http://csda.gencat.cat/web/.content/home/consell_superior_d_avalua/pdf_i_altres/static_file/quaderns-31.pdf

- Mobile Applications to train with math:

http://toolbox.mobileworldcapital.com / apps / android / attribute [TaxonomyTaxonomyValue4]: 134

- Main Geogebra's website in Catalunya:

  https://www.geogebra.org/

- Two Montessori videos:

  https://www.youtube.com/watch?v = eqe2NrjFQuE

  https://www.youtube.com/watch?v=kKpBj4o9gmo&authuser=0

6. **Do teachers use problem-based learning?**

100% of respondents say that they use it, although with different frequencies.

7. **Do you have the opportunity to develop lessons based on real-life applications?**

All respondents use it, although half of them found certain difficulties on how to better develop or change it.

8. **What examples are there of real-life applications for mathematics teaching?**

Examples:

- Perform mathematical exercises in everyday activities: setting the table, finding out who came to class today, organizing class tables, etc.
- Work with the calendar: what day is it today, temporal consciousness countdowns to a special day, etc.
- Measure their school in different ways.
- Mathematics and kitchen: buying food, figuring out what and how much they need, calculating portions and proportions, etc.
-

9. **What connections do you see as a mathematics teacher with teachers of other subjects? Which subjects are these?**

At this point we see very significant differences among respondents: five of the nine respondents said that they see connections of mathematics with other disciplines, highlighting Waldorf and Montessori (using music and movement, dance, drawing and crafts); while the remaining four do with little frequency or directly do not.

10. **What is the state of teachers' relationships with their pupils?**

Primary teachers in Spain teach mathematics as well as all of the other subjects, so this question does not apply to their "role" within the school. On the other hand, six of nine of our respondents see their role as someone who accompanies students in their pursuit of experiences from curiosity to creativity, not to seek answers but to develop questions.

11. **What domains of mathematics and science & technology are you as a teacher particularly interested in?**

We found three elements with a clear consensus among our respondents: that classes have a useful component that helps them solve things; that pedagogically is accompanied by visual elements, and finally it has always implied, directly or indirectly, a challenge that makes them persevere.

12. **What gaps would you perceive in capacity and strengths among students?**

Technically we found that many students have difficulty with trigonometry and fractions. At the competence level, the management of frustration is an important point because often students do not understand mathematics.

13. **What kind of extra challenges might be included to engage and stretch students?**

Most of the efforts of teachers to further motivate their students are closely related to teaching mathematics in a more visual, fun way and to have applicable meanings in their lives. Joan Bassas, one of our professionals, said, **"***We must provide our students an individualized space so that students can really focus. There must be moments of silence and pleasure.*"

14. **Do you think that your pupils see STEAM subjects as useful in everyday life?**

Here we find a significant lack of consensus: there are students who do see the usefulness of what they have learned, while there are also many others who do not see the applicability in everyday life. 50% of respondents have told us that their students are constantly asking about the usefulness and applicability of what they are learning.

15. This project plans to develop a toolkit for teachers, what would be useful to you in such a toolkit?

Below we have summarized several of the various proposals:

- Digital tools to manipulate objects (GeoGebra). Two of our respondents believe that digital materials would be a useful addition to the toolkit.
- Stories to make sense of what students have learned.
- Basic materials with instructions to work on the computer to discover an enigma.
- Tools to help collectivize learning of mathematics, which often are taught mostly individually.
- Exercises and proposals to relate mathematics to other disciplines (cooking, music, history, movement, etc.)
- Proposals to teach mathematics, not only cognitively, but physically and emotionally.
- We emphasize that the majority would use search engines for resources, because there currently are many resources on the Internet. The problem, though, is with the selection of these resources.

Existing Research findings on Maths Teaching: Spain

We share some recent research on the teaching of STEM from two European organizations working in this field (**Scientix** and the **International Journal of STEM Education**), and finally an article about developmental stages and important periods of cognition.

1) Scientix

Scientix promotes and supports a Europe-wide collaboration among STEM for teachers, education researchers, policymakers and other STEM education professionals: http://www.scientix.eu/web/guest

In the first conference of this organization, in 2011 in Brussels, the main idea is that the role of science education in tackling current societal problems, the EU's Europe 2020 strategy, cross-border collaboration, school curricula, assessment models, learning resource repositories, teacher training: these are some of the topics – a snapshot – of what was discussed at the Scientix European Conference, on 6-8 May 2011 in Brussels.

Within this conference, we highlight the conclusions of Marc Durando (Executive Director of European Schoolnet, Belgium):

"In most European countries the educational system is built on three pillars: the first pillar is the curriculum, and the teachers are the leaders of change in the curriculum in the light of the various developments at stake in various countries at the moment. The second pillar is the initial training of teachers, in-service training of teachers, inquiry-based techniques, etc. And the third pillar is assessment. This is where there will probably be a big change in the next ten years. There is a need to revise our assessment models, and that is a change where teachers should have a significant role to play. Teachers will be the driving force on these three pillars in order to make change happen.

Some keywords that came up in many of the presentations at the conference were ***motivation, empowerment, creativity, innovation*** *(…) Creativity and innovation in education are also essential elements which do not exclusively belong to the higher education sector or the research labs in universities. Innovation in education is also what teachers at primary and secondary levels do in their classes. They are defining a new pedagogy and they're also the owners of the innovation on a daily basis".*

http://files.eun.org/scientix/Gras-Velazquez-2012-Scientix_Proceedings_of_the_conference_final-inhouse-design.pdf

We also highlight, likewise, the conclusions of Gilles Laroche (Head of Unit, Directorate General for Research and Innovation, European Commission).

"Innovation is a complex process, and the success of new technologies which require heavy investments in time, human resources and finance is difficult to foresee, while the societal perceptions and impact are even more difficult to predict. Thus, after years of research financed by the Science in Society programme, and under the framework of the Innovation Union flagship, it has become clear that successful innovations need responsibility. This is why many efforts are focussed on the development of the Responsible Research and Innovation framework. To accomplish this, ***six action lines (AL) were developed:***

> ***A1L: Inclusiveness.*** *Societal challenges can only be achieved by involving all actors at an early stage: researchers, civil society organizations, industry and policy makers.*
>
> ***AL2: Gender equality in research content and in careers.*** *All actors should be on board: women and men.*
>
> ***AL3: Science education.*** *There will be no innovations without science education. It is not only important to Europe to increase the number of researchers, it is also crucial to build up a scientifically literate society to allow full participation in the research and innovation process.*

A4L: To promote open access. *This means giving free online access to peer-reviewed publications and in some cases to data. This will not only boost innovation, it will help creating the transparency needed between actors. This will be for the benefit of all: teachers, students, citizens.*

AL5: Ethics. *In order to respond adequately to societal challenges, research and innovation must include fundamental rights and ethical norms.*

AL6: Governance. *Through this AL we are developing models for responsible research and innovation".*

2) International Journal of STEM Education

The *International Journal of STEM Education* is a multidisciplinary journal in subject-content education that focuses on the study of teaching and learning in science, technology, engineering, and mathematics (STEM). https://stemeducationjournal.springeropen.com/

Some interesting articles from this site: Assessing teacher education and professional development needs for the implementation of integrated approaches to STEM education:

https://stemeducationjournal.springeropen.com/articles/10.1186/s40594-017-0068-1

The effects of an afterschool STEM program on students' motivation and engagement:

https://stemeducationjournal.springeropen.com/articles/10.1186/s40594-017-0065-4

3) Article about Developmental stages and important periods of probability cognition in 6 to 14 year-old students (Zikun Gong and Shengqing He):

In this study, 906 students from 6 to 14 years of age were selected and studied in the stages of development and the periods of probability cognition. The study shows that probabilistic cognition of students from 6 to 14 years experiences the following 5 stages: slow development I (6-7 years), rapid development I (8-9 years), slow development II (10 years), development Fast II (11-12 years) and consolidated phase (13-14 years). In addition, there are two important periods in students' cognitive development: the first at 8-9 years of age and the second at 11-12. Even in the highest

developmental stage, students can understand the numerical representation, the probability distribution and the fractional representation while the level of mastery can not be reached, which suggests the limitation of the probabilistic cognition of the students. Consequently, the curriculum must take into account the level of cognitive development of the students and establish reasonable cognitive objectives.

https://dialnet.unirioja.es/servlet/articulo?codigo=6168893

Mathematics Education in The Netherlands

Background Research

Main Source of Information on Attainment Goals, mathematics and technology in particular:

Tule Slo: http://tule.slo.nl/RekenenWiskunde/F-KDRekenenWiskunde.html

Primary Level

Mathematics in Dutch Primary Education, Ministry of Education, Culture and Science (2006) Attainment targets primary education (Project number: 36027/8000). Consulted via Source :http://www.slo.nl/primair/kerndoelen/Kerndoelenboekje.pdf/

The aim of mathematics in primary education in the Netherlands is that pupils, in the context of meaningful situations, begin to feel familiar with numbers, sizes, forms, structures within appropriate relationships and procedures (Ministry of Education, Culture and Science, 2006). This means that pupils learn to use ''maths language'', become mathematically and numerically literate.

The pupils should not only be able to apply mathematics within the subject itself, but also daily life must be involved, as well as the integration with other subjects. In that perspective, the teacher must take the following aspects into account when presenting the mathematical content:

- Prior knowledge
- What pupils need to learn within the schedule/learning goals
- The interests of the pupils.

In this way the pupils will receive education at the right level, the pupils will have fun and the pupils will be challenged. In addition, the pupils not only have to learn individually but also in groups, in that way; they can ask mathematical questions, they can formulate problems and solve problems together so they practise their skills for; formulating, noting, giving feedback to each other, organizing and developing.

The learning goals for primary education, for mathematics in the Netherlands are expressed in eleven specific goals, which all have been worked out in so called learning trajectories http://tule.slo.nl/RekenenWiskunde/F-KDRekenenWiskunde.html

Science and Technology

Platform Education 2032 have also created descriptions for education in the Netherlands, with goals for the year 2032; here they also pay attention to Science and Technology. Both reports pay attention to a coherent and interdisciplinary form of education, with the actuality as starting point for education and with attention to connections between subjects. The same goes for the following skills; creating, critical thinking, problem solving and working together. (http://onsonderwijs2032.nl/wp-content/uploads/2016/01/Ons-Onderwijs2032-Eindadvies-januari-2016.pdf)

The reason for this learning framework for Science and Technology is that good education in technology is needed to give pupils insight into the meaning of technology in their lives, with the hope that they will opt for a technical training in the future.

Since 2005 there is a change in the vision on technology in education. First (in the Netherlands), technology was meant to teach pupils how to create things, so they learned about materials, tools and technical insights. Since 2005 pupils also are to develop an interest in science key concepts, and also that technology education teaches them to discover and explore competencies. However, the number of schools that actually works with this new vision since 2005 is limited. This is because there is a focus on language and mathematics and less attention to history, geography and science.

For that reason, platform Beta Techniek appointed together with the PO-council (in 2012) a committee to explore possibilities. The goal of this committee was to explore the possibility to implement science and technology in primary education by 2020. The final advice of this committee was: SLO has to develop a curriculum with clear learning trajectories and examples for science and technology in primary education. Based on this report questions were asked in parliament. These questions in

parliament led to an official order from the ministry of education to SLO, for developing the curriculum. Also, a technical pact has been concluded: National Techniekpact 2020 (https://www.techniekpact.nl/)

Netherlands: Interview Results

Section 1: Country Context

1. **What is your opinion of the Maths curriculum in your country?**

The Dutch curriculum is textbook-oriented, highly structured and has lots of attention to different levels of achievement. Two streams: realistic (contextualized) mathematics on the one hand and the more traditional approach of rote-learning and memorizing on the other.

2. **What are the positive aspects?**

The textbook gives support and the possibility to differentiate but in the end the teachers remain the determining factor. More and more teachers are engaging with projects next to the set program. The continuous learning trajectories guarantee the level and quality.

3. **What are the negative ones?**

The testing culture - besides children don't get what they deserve because they have to repeat too much or have to slavishly follow the line of the textbook.

4. **Do you have the option to deliver the curriculum in your own way, for example are there proscribed class times, restrictions due to state examinations?**

Yes, there is enough freedom to arrange lessons according to the teacher's wishes and ideas. However, each school day should contain 45-60 minutes of maths teaching/learning. Teachers should also always observe the objectives and attainment goals. Teachers do not often develop their own lessons due to time pressure; teachers indicate they just do not have the time to do so.

Section 2: Schools

1. **How does your school support mathematics and related subjects?**

This varies. Mathematics is a spearhead, so that extra attention is paid to this subject. Teacher educators try to transfer a sense of enjoyment of maths and introduce various forms of arithmetic and maths so that student teachers can make informed choices. Besides, schools have a maths coordinator who informs and supports.

2. **Do school timetables provide sufficient time for mathematics?**

One hour a day.

3. **Do schools provide adequate facilities and resources for the teaching of maths and sciences? For example, is it equipped in science labs? If so, how are they organized? What kind of access students have (only during the classes, afterschool etcetera?). Which tools can be found in the labs?**

In general, we have the following facilities in the Netherlands: computers, books, standard materials such as measuring cups, blocks, calculators, rulers, abacus. Some schools have project rooms where e.g. parents give special workshops such as 3d printing, Lego League, robotics etc. The teacher training department organizes a maths contest every year for which students are encouraged to bring authentic materials such as newspaper clippings. Other schools work with a method called "top entrepreneurs".

Section 3: Teachers

1. **What kind learning of materials are used most often in classroom (books, websites, mobile apps)?**

Textbooks, clocks, fake money, paper, hourglasses, scales, building blocks, abacus. Most Dutch schools have tablets/I-pads (Snappet, maths garden, Ambrasoft, Squla)

2. **Do you have any preferences? Please name one favorite tool that you use or know is used.**

Depends on the class, the group and the level. In general, the textbook is leading. Besides internet-based activities. Authentic where possible and easily applicable. Many teachers indicate they like an online course, because it is easier to differentiate and the check-yourself option saves the teacher a lot a time.

3. **What styles of teaching are offered? Describe the interactions in a typical class.**

In the Netherlands, we work with the direct instruction or IGDI model. This method has fixed moments of interaction; it is based on "frontal", teacher-oriented teaching. Some teachers give pupils in the higher forms more freedom. In the Netherlands, we have many different school philosophies (e.g. Dalton, Montessori, Jena) where pupils can join scheduled instruction moments when they feel the need. We do have the national Cito test in the course of the final year of all PE and which determines the content of the lessons.

4. **What are the typical teaching methodologies currently employed by you?**

Direct Instruction mixed with moments of interactivity and individual/groupwork

5. **Do teachers use problem-based learning?**

In regular education teachers do not commonly use STEAM or problem-based learning. They indicate that they would like to but just don't have enough time. They do apply these concepts during projects or the Big Maths Day. One teacher indicates she does try to apply STEM on a modest scale by having pupils work on problems in pairs, for which they have to develop a strategy together. She does so because this is the way they will have to learn in further education.

6. **Do you have the opportunity to develop lessons based on real-life applications?**

Yes, teachers have that freedom, however they indicate they do not have enough time.

7. **What examples are there of real life applications for mathematics teaching?**

The schoolyard, measures and weights, recipes, maps, all spaces in the school, a newspaper article, participating in the Big Maths Day, authentic examples such a home renovation of the teacher, a renovation of the school. For science: experiments with everyday substances (found in the home, the garden or the kitchen)

8. **What links do you as mathematics teacher perceive with teachers of other subjects? Which subjects are these?**

In the Netherlands, one teacher teaches all subjects. Lessons are generally not prepared together in collaboration with colleagues. Ideas are shared during team meetings. Projects - in contrast - are prepared jointly. The maths coordinator keeps the team updated and in some cases the maths coordinator pays schools visits.

9. **What would be the main challenges in these domains/subjects?**

As to 21st century skills: pupils still have to learn to solve problems and to think critically. However, teachers must also become more competent in this area, which will take time since teachers are stuck in their own routines. Another challenge is to reduce the testing culture. Besides we have to find a balance between authentic maths and old-fashioned rote-learning (which in some case is simply necessary). As to science and technology: the challenge is getting these into the curriculum and to supply the teachers with enough knowledge and skills to teach these subjects.

10. **What gaps would you perceive in capacity and strengths among students?**

Pupils are not so good at memorizing/automatizing anymore. But also solving problems is hard for them, depending on the group and level. As to measurements, fractions and geometry: some claim that pupils are not yet ready to make the connection to reality yet.

11. **What and where could extra support be provided?**

Pupils need more and better mentoring when it comes to acquiring metacognitive skills. Besides, pupils need concrete material so that they know what they are really doing. Teachers should first work on their own skills; check the entire program at the

start of the schoolyear, determine where the gaps are and develop lessons themselves to cover those gaps. Teachers should have more time for professional development.

12. **What kind of extra challenges might be included to engage and stretch students?**

Encourage pupils to collaborate more, reward them by giving them credits or points on the basis of their level. Make maths more practical and applied. More focus on thematic and research-based work-forms. Another expert calls for a pedagogy that gives pupils insight into what they haven't mastered yet so that they can set targets and on the basis of that become motivated to work towards their own goals. For future teachers, it is important to acquaint them with more with an integration of subjects and to stimulate them to bring the real world into the classroom.

13. **This project plans to develop a toolkit for teachers, what would be useful to you in such a toolkit?**

For the teacher, a practical manual to the assignments so that a substitute or temporary teacher can also work with the toolkit. A clear indication of what the connection is with the attainment goals. Supply Didactics and subject specifics. Simple instructions and tips for the teacher that can be applied without too much study. For the pupils, a scheme or framework so that they can see the larger picture. Pupils need freedom within a certain amount of structure. Although there are also voices that say that pupils should be forced to really think about certain problems. A number of good math apps, theme suggestions to link to activities. 30-minute activity cards (indicating materials and objectives of the assignment). One of the experts indicates that he would like to see research oriented tasks in the toolkit since that is a development in the Netherlands that we will hear more of.

Section 4: Pupils

1. **What is the state of teachers' relationships with their pupils?**

The distance between pupils and teacher is relatively small in the Netherlands. Some teachers indicate that we are possibly taking this a bit too far. The experts indicate

that there is a good balance between autonomy, competence and relationship. Generally speaking, the teacher is the authority in class, although there is a tendency towards a more coaching than leading role.

2. **What Domains of mathematics and science & technology is particularly interesting for your students?**

Something real and authentic such as planning a school trip. Topics such as currency and programming. Another often heard view: what the teacher finds interesting and mentions a lot becomes automatically interesting to the pupils and leads to higher scores.

3. **Which of the learning methodologies motivates pupils the most? Please give concrete examples: games, experiments, artistic performances, platforms, books etcetera. If there are any particularly useful links/site/resources please provide.**

Varies per pupil. There are pupils who learn by trial and error. Other pupils want the teacher to tell them what to do. Pupils like games but if they play them too often the motivation decreases. Also, teachers should motivate, connect to the world of young kids, no matter what textbook materials you use. Gamification is mentioned. In general teachers and experts say it depends on the group, their learning styles and level.

4. **Do you think that your pupils see STEAM subjects as useful in everyday life?**

No consensus here: some teachers indicate that pupils see the use but not of all domains, others indicate that it takes the teacher to continuously explain and give examples.

5. **What feedback on STEAM do you receive from your pupils?**

   n/a

INTERVIEWS

Frits Barth,

Kush Schoonhoven

Andreja Laffra

Jelle Brandsma

Harm Veenstra

Age Wesselius

Ronald Keijzer

Jurjen Uiterdijk

Interviewer: Harma de Vries

Leeuwarden May 2018.

Existing Research findings on Maths Teaching: the Netherlands

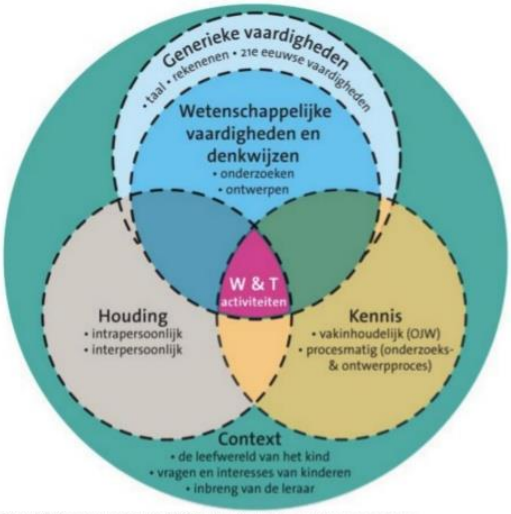
According to Graft, Klein Tank and Baker (2016) the definition for education and technology is: It's a way of looking at the world, through wondering questions and problems will arise and then the search for answers is about to start, answering the questions and problems which leads to solutions in the form of knowledge and products. These solutions designed by the learners form the starting point for new questions. Education in science and technology will stimulates and consolidate a curious and questioning, a problem solving attitude with children. It's about enquiry and design based education, which will be developed together with the 21 century skills such as ; creativity, entrepreneurship, critical thinking, being able to coöperate and ICT literacy; in short, it will teach pupils about the world.

According to the learning framework Science and Technology, science and technology go together with integration between subjects. Because learners are able to apply science and technology activities, knowledge and attitudes they have acquired in the language class and mathematical lessons but also within geography, history and physics. The writers worked out the integration of science and technology and mathematics in chapter 4, a concrete example is in table 17. Graft, Klein Tank en Beker (2016)

Science and technology have been worked out in learning trajectories; these are built up from attainment goals, intermediate goals in three phases: initially, then and finally. No distinction has been made between groups or age (like SLO does), because they think it's dependent on the (school) situation and the development of the children. If you would like to use age classifications, they advise: group 1+2 = initially, group 3- max 6 is then and group 7-8 is finally.

In chapter 3 we get a description of how this new form of science and technology differs from the current education; Children live in a knowledge intensive society, where knowledge is developing fast. That's why children need skills and an attitude to become flexible, so they can handle this knowledge. These skills, attitude and knowledge have to be part of the available education. That's why a model with science and technology in cohesion has been created.

Central in the model is science and technology in which the three aspects come together; skills, knowledge and attitude. Included are such skills as; observing, measuring, using different ways of thinking, materials, tools, reflecting and evaluating the process. These skills may depend on the subject, receive a personal interpretation or more domain generic if it's about language, mathematics or 21st century skills. The research and design skills ask for a certain attitude, which is reflected in: being curious, being critical, open-minded and creative. About the knowledge aspect: it may be about animals, history, social geography etc.



Figuur 1. De componenten van W&T-onderwijs en hun onderlinge samenhang

Other sources that could be interesting:

https://www.ipabo.nl/upload/publicaties/Ass%20Lectoraat%20W&T/vb_36_5_o_en_o_Hotze_Samenhang_tussen_rekenen-wiskunde_en_wetenschap_en_technologie.pdf

It's an article from a journal for teachers in primary education, about mathematics. In this article they discuss the implementation of science and technology in primary education, indicating that this is difficult. They also write about the lessons in mathematics, that they need a more investigative way of learning within a meaningful context. A reason for them to talk about the similarities between both subjects and how you could integrate both subjects. The article is in Dutch.

https://nvvw.nl/2017/10/10/rekenen-in-het-regeerakkoord/

This is the partnership agreement of the government (2017). In short they write about education: Teachers, together with pupils and parents need to revise the curriculum as of 2019. With this agreement the following subjects will be better prepared for the future; Dutch language and mathematics. digital literacy, practical skills, technology skills and citizenship.

http://onsonderwijs2032.nl/wp-content/uploads/2016/01/Ons-Onderwijs2032-Eindadvies-januari-2016.pdf

About what the pupils need to learn in 2032 in primary education.

https://www.techniekpact.nl/nationaal-techniekpact-2020

English report:

https://www.techniekpact.nl/cdi/files/e3bd421f98a0f362b6a13091de60d08978df34e9.pdf

Goals National Techniekpact2020, according to the summary: Continuous interaction between the education community and business sector will be crucial in ensuring the future availability of well-trained engineers. This interaction will help ensure qualitative, up-to-date education that reflects current labour market demands. The twelve objectives, which have been partially reformulated and aligned with existing agreements, offer all partners a basis upon which to continue and initiate relevant measures, achieve results and engage in further collaborations over the coming years.

SUMMARY

This document has provided an overview of the issues and themes in mathematics and STEM teaching, with extensive reference to the conditions and experiences in the four partner countries; Poland, Ireland, Spain and the Netherlands.

The final section is a contribution from the international STEAM expert, Dr Cristiana Assumpção. For many years Dr Assumpção headed the STEAM Department of Colegio Bandeirantes in São Paulo, Brazil. She is now Education and Technology Director for EDUC4X100 in the United States. She has contributed her set of preferred readings and resources to the project .

ADDED VALUE PROJECT RESOURCES

(books and materials that inspired me to create the STEAM curriculum)

Cristiana Mattos Assumpção Ed.D.

Materials

Invent to Learn (book and website) - https://inventtolearn.com/ - bringing the world of making to the school.

Tony Wagner's books:

- Creating Innovators (http://www.tonywagner.com/creating-innovators/)
- Most Likely to Succeed (http://www.tonywagner.com/most-likely-to-succeed-preparing-our-kids-for-the-innovation-era/) - Tells the story of High Tech High in California and how they created a culture of innovation.

The Innovators Mindset

https://www.amazon.com/Innovators-Mindset-Empower-Learning-Creativity/dp/0986155497

George Couros is one of the most forward thinking educational leaders I see today. He is doing a great job of preparing teachers and administrators to rethink how they are doing education and putting the learner in the center of the process.

Learner Centered Innovation

https://katielmartin.com/2018/02/06/learner-centered-innovation/

Katie Martin works closely with George Couros and has written a wonderful book on how to truly bring innovation to the school. I am taking their course online and reading her book thoroughly, and she describes exactly what we had to go through to really innovate.

From STEM to STEAM

https://www.amazon.com/STEM-STEAM-Brain-Compatible-Strategies-Integrate/dp/1452258333/ref=mt_paperback?_encoding=UTF8&me=

Sousa and Pilecki explained in the light of neuroscience the importance of bringing the Arts to STEM. Much of what is said for Arts also helps in Math if you think about working logic and calculations.

Lifelong Kindergarten

https://mitpress.mit.edu/books/lifelong-kindergarten

Written by Mitchel Resnick of the Lifelong Kindergarten group at the MIT Media Lab, they were close partners with us to develop the STEAM curriculum and we followed many of the ideas shared in this book. He makes the case for Creative Learning in schools.

Meaningful Making

http://fablearn.stanford.edu/fellows/page/meaningful-making-book

Compilation of projects being done by researchers at Stanford.

Grit

https://angeladuckworth.com

This book is closely related to the Social Emotional Learning as you talk about teaching kids perseverance.

Needs Analysis

Juventude Conectada Report by the Telefonica Vivo Foundation in Brazil - http://fundacaotelefonica.org.br/projetos/juventude-conectada/

This report presents the results of a research with youth in Brazil in relation to their connectivity and use of the internet. It gives a good idea of the profile of the young user today, which can help think about materials and inform decisions. The second edition can be found here - http://fundacaotelefonica.org.br/wp-content/uploads/pdfs/Juventude-Conectada-2016.pdf

Horizon Project

https://www.nmc.org/nmc-horizon/

The Horizon Reports are organized by regions and sectors. They analyze yearly the trends, challenges and technologies most likely to widely impact education in the near future. I participated in their advisory board for the K12 sector from 2010 to 2017 (when they went into bankruptcy unfortunately). But CoSN is taking over. It's an amazing resource as it is the fruit of research and discussion of educational technology leaders from all over the world.

Research Findings

SEL (Social Emotional Learning) Handbook - https://www.guilford.com/books/Handbook-of-Social-and-Emotional-Learning/Durlak-Domitrovich-Weissberg-Gullotta/9781462527915/reviews

This book reviews the impact of different interventions in the different sectors of education. Not only does it inform on what worked best, but also on the different tools they used to analyze the impact. A great resource for this research.

TOOLS

NSTA books

http://www.nsta.org/publications/press/

We used several tools presented in these books in our Science and STEAM courses. There are several that share formative assessment tools such as probes to find out how students are thinking and to make thinking visible for them. The probes can be found in the Uncovering Student Ideas series by Page Keeley.

CASEL

http://casel.org

Resources on Social Emotional Learning.

BIE

https://www.bie.org

Best recommended site on Project Based Learning. Got a lot of resources from here.

Mindmapping

https://www.makeuseof.com/tag/8-free-mind-map-tools-best-use/

When thinking about using Design Thinking and other visible thinking tools, mindmaps are a great way to go! I am the hugest fan of teaching students how to use mindmaps to organize their ideas. I personally used a paid one called iThoughts.

Design Thinking for Educators

https://www.ideo.com/post/design-thinking-for-educators

We used this manual as a reference when planning our own design thinking lessons. We used many of the tools here.

Seesaw

https://web.seesaw.me

For documentation. You can organize the kid's production and share easily. Finding a good documentation tool was a challenge for us. We were using Exchange so we tried to use collaborative OneNote notebooks, but the learning curve was too high. I have found that working with shared folders in Google Drive tends to work better with teachers and students. They naturally gravitate towards those tools. Here in the US I have seen many teachers using this tool, Seesaw, which I tried out and really liked, and is very easy for kids to use. It has several interesting resources.

APPENDIX

AV Project Current Innovations in Maths Teaching

Current innovative programs using Problem-Based Learning, Real Life applications Located and examined, presented among all partners.

1. Columbia University: innovative programme for educators https://earth2class.org/site/
2. https://www.ad.nl/binnenland/alle-friese-basisscholen-krijgen-een-robot~a915d3bc/
    a. In 2017 all primary schools in Friesland received a robot. In this way the schools have the opportunity to learn and teach how to program so that the robots can for example: walk, dance and talk. In that way the schools and high tech companies want to make children enthusiastic for technology.

3. Illuminations is a project designed by the National Council of Teachers of Mathematics (NCTM), that works to serve maths teachers by increasing access to quality standards-based resources for teaching and learning mathematics, including interactive tools for students and instructional support for teachers. https://illuminations.nctm.org/Default.aspx

4. Resources and activities for the teaching of mathematics in the classroom in a manipulative way: https://www.math.nmsu.edu/~breakingaway/

5. National Library of Virtual Manipulatives for Interactive Mathematics: http://nlvm.usu.edu/ Page from the University of the State of Utah that provides online resources for teaching mathematics. The contents are grouped by school levels and by subject.

6. Spanish company of pedagogical tools for maths (in Spanish): https://www.smartick.es

7. Blog divulging innovative math tools in primary school (in Spanish): https://matematicaenprimaria.com/

8. Geogebra Association, a program to work on geometry: http://acgeogebra.cat/ and Geogebra, a program for geometry, in English version: https://www.geogebra.org/

9. Claudi Alsina, professor of mathematics, researcher and disseminator: http://claudialsina.com/ca/

10. Interview with Anton Aubanell, professor of mathematics education at the University of Barcelona. He makes several very interesting experiments to explain certain aspects of mathematics and geometry: http://www.rtve.es/alacarta/videos/para-todos-la-2/para-todos-2-entrevista-anton-aubanell-ensenar-matematicas/2508808/
11. Program and mobile app. Useful tool" "scan a math problem for an instant result" https://photomath.net
12. Help in math problems, information on mathematical subject or topics http://www.wolframalpha.com

13. innovative programme for teacher (in polish): http://www.ibe.edu.pl/pl/babel

14. Educational resources aligned to Catalan math curriculum: http://edu365.cat/

15. Online listing of educational apps ranked by teachers:
16. http://toolbox.mobileworldcapital.com/apps/android/metacategory:Aprendre_angles

17. Catalan museum of mathematics: https://mmaca.cat/

18. **"First, manipulate. Then, draw and then, the symbols".** More than twelve public, concerted and private Spanish schools of several autonomous communities decided to put into practice in their classrooms during the 2016-2017 academic year the mathematical **method of Singapore**, which has placed this Asian country at the head of the international reports of the OECD. The Singapore method is based primarily on Bruner's 'three-step learning': ***"Children must first manipulate, then draw, and only then move on to traditional mathematical symbols."*** http://www.publico.es/sociedad/mejor-manera-ensenar-matematicas-colegio.html
19. **The Singapore method.** Yeap Ban Har is an expert math teacher from Singapore who has become a world celebrity for finding a teaching method that puts his country as the best student in mathematics. ***"All learning begins in a concrete way, then pictorial and finally abstract,"*** he explains. It also applies the **theory of the spiral, which involves trying to reach the same place through different paths, without repeating or memorizing a single way** as they do in the classrooms of half the world.
20. https://politica.elpais.com/politica/2017/06/19/diario_de_espana/1497892421_568601.html
21. STEAM RESEARCH CENTRE Integrated STEAM education is defined as the interdisciplinary teaching and learning of Science, Technology, Engineering, Art and Mathematics (STEAM). The mission of the STEAM Research Centre is to promote the synergy of these areas of learning in an educational context, with the development of resources, concepts of pedagogical practice, reviews and evaluations set within a research-informed and research-driven framework. The mission is founded on the research team's understanding of the need to

enthuse and empower future generations as effective innovators, contributing to the wider society and economy through the study and application of STEAM education. A joint initiative of St Mary's College and Stranmillis College in Belfast. Dr Kieran McGeown has contributed to the Added Value project https://www.smucb.ac.uk/academic/research/researchcentres_csteam.asp?cid=14151203967

22. PISA Mathematics: A Teacher's Guide, Department of Education and Science (Dublin, 2007). Dr Elizabeth Oldham, et al.